

14. Неравенства

Блок 1. ФИПИ (www.fipi.ru) + другие источники

1) Показательные неравенства

Задание 1. Решите неравенство:

- 1) $12^x - 8^x - 2 \cdot 6^{x+1} + 3 \cdot 4^{x+1} + 32 \cdot 3^x - 2^{x+5} \leq 0$;
- 2) $63^x - 27^x - 12 \cdot 21^x + 12 \cdot 9^x + 27 \cdot 7^x - 3^{x+3} \leq 0$;
- 3) $45^x - 27^x - 18 \cdot 15^x + 2 \cdot 9^{x+1} + 81 \cdot 5^x - 3^{x+4} \leq 0$;
- 4) $28^x - 8^x - 16 \cdot 14^x + 4^{x+2} + 64 \cdot 7^x - 2^{x+6} \leq 0$;
- 5) $27 \cdot 45^x - 27^{x+1} - 12 \cdot 15^x + 12 \cdot 9^x + 5^x - 3^x \leq 0$;
- 6) $2 \cdot 20^x - 2 \cdot 8^x - 17 \cdot 10^x + 17 \cdot 4^x + 8 \cdot 5^x - 2^{x+3} \leq 0$.

Задание 2. Решите неравенство:

- | | |
|---|---|
| 1) $\frac{1}{5^x + 31} \leq \frac{4}{5^{x+1} - 1}$; | 9) $\frac{3}{(2^{2-x^2} - 1)^2} - \frac{4}{2^{2-x^2} - 1} + 1 \geq 0$; |
| 2) $\frac{1}{2^x + 21} \leq \frac{3}{2^{x+2} - 1}$; | 10) $\frac{16}{(3^{2-x^2} - 1)^2} - \frac{10}{3^{2-x^2} - 1} + 1 \geq 0$; |
| 3) $\frac{2}{7^x - 7} \geq \frac{5}{7^x - 4}$; | 11) $\frac{15}{(4^{2-x^2} - 1)^2} - \frac{16}{4^{2-x^2} - 1} + 1 \geq 0$; |
| 4) $\frac{4}{3^x - 3} \geq \frac{5}{3^x - 2}$; | 12) $\frac{105}{(2^{4-x^2} - 1)^2} - \frac{22}{2^{4-x^2} - 1} + 1 \geq 0$; |
| 5) $\frac{3^x - 1}{3^x - 3} \leq 1 + \frac{1}{3^x - 2}$; | 13) $\frac{15^x - 3^{x+1} - 5^{x+1} + 15}{-x^2 + 2x} \geq 0$; |
| 6) $\frac{6^x - 1}{6^x - 6} \leq 1 + \frac{3}{6^x - 4}$; | 14) $\frac{14^x - 7^{x+1} - 2^{x+1} + 14}{-x^2 + 4x} \geq 0$; |
| 7) $\frac{4^x - 2}{4^x - 3} \leq 1 - \frac{1}{4^x - 4}$; | 15) $\frac{10^x - 25 \cdot 2^x - 2 \cdot 5^x + 50}{5x - x^2 - 4} \geq 0$; |
| 8) $\frac{5^x - 4}{5^x - 3} \leq 1 - \frac{3}{5^x - 5}$; | 16) $\frac{12^x - 16 \cdot 3^x - 3 \cdot 4^x + 48}{6x - x^2 - 5} \geq 0$. |

Задание 3. Решите неравенство:

- | | |
|--|--|
| 1) $\frac{13 - 5 \cdot 3^x}{9^x - 12 \cdot 3^x + 27} \geq 0,5$; | 4) $\frac{2^x}{2^x - 3} + \frac{2^x + 1}{2^x - 2} + \frac{5}{4^x - 5 \cdot 2^x + 6} \leq 0$; |
| 2) $\frac{7 - 2 \cdot 2^x}{4^x - 12 \cdot 2^x + 32} \geq 0,25$; | 5) $\frac{5^x}{5^x - 4} + \frac{5^x + 5}{5^x - 5} + \frac{22}{25^x - 9 \cdot 5^x + 20} \leq 0$; |
| 3) $\frac{31 - 5 \cdot 2^x}{4^x - 24 \cdot 2^x + 128} \geq 0,25$; | 6) $\frac{3^x}{3^x - 3} + \frac{3^x + 1}{3^x - 2} + \frac{5}{9^x - 5 \cdot 3^x + 6} \leq 0$. |

Задание 4. Решите неравенство:

$$1) \frac{6^x - 4 \cdot 3^x}{x \cdot 2^x - 5 \cdot 2^x - 4x + 20} \leq \frac{1}{x-5};$$

$$5) \frac{9^x - 2 \cdot 3^{x+1} + 4}{3^x - 5} + \frac{2 \cdot 3^{x+1} - 51}{3^x - 9} \leq 3^x + 5;$$

$$2) \frac{15^x - 9 \cdot 5^x}{x \cdot 3^x - 4 \cdot 3^x - 9x + 36} \leq \frac{1}{x-4};$$

$$6) \frac{4^x - 3 \cdot 2^{x+2} + 5}{2^x - 3} + \frac{3 \cdot 2^{x+2} - 4}{2^x - 4} \leq 2^x + 3.$$

$$3) \frac{25^x - 5^{x+2} + 26}{5^x - 1} + \frac{25^x - 7 \cdot 5^x + 1}{5^x - 7} \leq 2 \cdot 5^x - 24;$$

$$4) \frac{4^x + 2^{x+1} - 36}{2^x - 5} + \frac{4^{x+1} - 2^{x+5} + 4}{2^x - 8} \leq 5 \cdot 2^x + 7;$$

Задание 5. Решите неравенство:

$$1) \frac{9^x - 3^x + 2}{9^x - 3^x} + \frac{5 \cdot 3^x - 19}{3^x - 4} \leq \frac{2 \cdot 3^{x+1} - 2}{3^x};$$

$$5) 125^x - 25^x + \frac{4 \cdot 25^x - 20}{5^x - 5} \leq 4;$$

$$2) \frac{4^x - 2^x + 1}{4^x - 2^x} + \frac{2^{x+1} - 9}{2^x - 7} \leq \frac{3 \cdot 2^x - 1}{2^x};$$

$$6) 8^x - 3 \cdot 4^x + \frac{9 \cdot 4^x - 288}{2^x - 9} \leq 32;$$

$$3) 2^x - 6 - \frac{9 \cdot 2^x - 37}{4^x - 7 \cdot 2^x + 12} \leq \frac{1}{2^x - 4};$$

$$7) \frac{4^x - 2^{x+3} + 7}{4^x - 5 \cdot 2^x + 4} \leq \frac{2^x - 9}{2^x - 4} + \frac{1}{2^x - 6};$$

$$4) 5^x - 24 - \frac{22 \cdot 5^x - 113}{25^x - 7 \cdot 5^x + 10} \leq \frac{1}{5^x - 5};$$

$$8) \frac{9^x - 10 \cdot 3^x + 9}{9^x - 4 \cdot 3^x + 3} \leq \frac{3^x - 11}{3^x - 3} + \frac{1}{3^x - 5}.$$

II) Логарифмические неравенства

Задание 6. Решите неравенство:

$$1) (\log_2^2 x - 2 \log_2 x)^2 + 36 \log_2 x + 45 < 18 \log_2^2 x;$$

$$2) (\log_2^2 x - 2 \log_2 x)^2 + 22 \log_2 x + 24 < 11 \log_2^2 x;$$

$$3) \frac{\log_4(16x^4) + 11}{\log_4^2 x - 9} \geq -1;$$

$$7) \frac{\log_4(64x)}{\log_4 x - 3} + \frac{\log_4 x - 3}{\log_4(64x)} \geq \frac{\log_4 x^4 + 16}{\log_4^2 x - 9};$$

$$4) \frac{\log_2(4x^2) + 35}{\log_2^2 x - 36} \geq -1;$$

$$8) \frac{\log_2(32x)}{\log_2 x - 5} + \frac{\log_2 x - 5}{\log_2(32x)} \geq \frac{\log_2 x^{16} + 18}{\log_2^2 x - 25};$$

$$5) \frac{\log_3(9x) - 13}{\log_3^2 x + \log_3 x^4} \leq 1;$$

$$9) \frac{\log_5(25x)}{\log_5 x - 2} + \frac{\log_5 x - 2}{\log_5(25x)} \geq \frac{6 - \log_5 x^4}{\log_5^2 x - 4};$$

$$6) \frac{\log_7(49x) - 3}{\log_7^2 x + \log_7 x^2} \leq 1;$$

$$10) \frac{\log_3(81x)}{\log_3 x - 4} + \frac{\log_3 x - 4}{\log_3(81x)} \geq \frac{24 - \log_3 x^8}{\log_3^2 x - 16}.$$

Задание 7. Решите неравенство:

$$1) \frac{\log_8 x}{\log_8 \left(\frac{x}{64}\right)} \geq \frac{2}{\log_8 x} + \frac{3}{\log_8^2 x - \log_8 x^2};$$

$$5) \frac{2\log_9(x^2 + 4x)}{\log_9 x^2} \leq 1;$$

$$2) \frac{\log_3 x}{\log_3 \left(\frac{x}{27}\right)} \geq \frac{2}{\log_3 x} + \frac{5}{\log_3^2 x - \log_3 x^3};$$

$$6) \frac{2\log_5(x^2 - 5x)}{\log_5 x^2} \leq 1.$$

$$3) 1 + \frac{5}{\log_4 x - 3} + \frac{6}{\log_4^2 x - \log_4(64x^6) + 12} \geq 0;$$

$$4) 1 + \frac{10}{\log_2 x - 5} + \frac{16}{\log_2^2 x - \log_2(32x^{10}) + 30} \geq 0;$$

Задание 8. Решите неравенство:

$$1) \log_2^2(x^2 - 9) - 9\log_2(x^2 - 9) + 20 \geq 0;$$

$$7) x^2 \log_{512}(x+7) \leq \log_2(x^2 + 14x + 49);$$

$$2) \log_3^2(x^2 - 16) - 5\log_3(x^2 - 16) + 6 \geq 0;$$

$$8) x^2 \log_{343}(x+3) \leq \log_7(x^2 + 6x + 9);$$

$$3) \log_5^2(25 - x^2) - 3\log_5(25 - x^2) + 2 \geq 0;$$

$$9) x^2 \log_{625}(6 - x) \leq \log_5(x^2 - 12x + 36);$$

$$4) \log_3^2(81 - x^2) - 7\log_3(81 - x^2) + 12 \geq 0;$$

$$10) x^2 \log_{512}(4 - x) \geq \log_2(x^2 - 8x + 16);$$

$$5) \log_4^2(64 - x^2) - 5\log_4(64 - x^2) + 6 \geq 0;$$

$$11) x^2 \log_{625}(-2 - x) \geq \log_5(x^2 + 4x + 4);$$

$$6) \log_7^2(49 - x^2) - 3\log_7(49 - x^2) + 2 \geq 0;$$

$$12) x^2 \log_{81}(-1 - x) \geq \log_3(x^2 + 2x + 1).$$

Задание 9. Решите неравенство:

$$1) \log_3((2-x)(x^2+5)) \geq \log_3(x^2-5x+6) + \log_3(4-x);$$

$$2) \log_5((3-x)(x^2+2)) \geq \log_5(x^2-7x+12) + \log_5(5-x);$$

$$3) \log_2^2(8+2x-x^2) + 9\log_{0,5}(8+2x-x^2) + 18 > 0;$$

$$4) \log_2^2(16+6x-x^2) + 10\log_{0,5}(16+6x-x^2) + 24 > 0;$$

$$5) \log_2^2(4+3x-x^2) + 7\log_{0,5}(4+3x-x^2) + 10 > 0;$$

$$6) \log_5^2(5+4x-x^2) + 4\log_{0,2}(5+4x-x^2) + 3 > 0.$$

Задание 10. Решите неравенство:

$$1) \log_5\left(\frac{2}{x} + 2\right) - \log_5(x+3) \leq \log_5\left(\frac{x+6}{x^2}\right); \quad 2) \log_3\left(\frac{1}{x} + 2\right) - \log_3(x+5) \leq \log_3\left(\frac{x+4}{x^2}\right).$$

Задание 11. Решите неравенство:

$$1) \log_7(2x^2+12) - \log_7(x^2 - x + 12) \geq \log_7\left(2 - \frac{1}{x}\right);$$

$$2) \log_3(x^2+2) - \log_3(x^2 - x + 12) \geq \log_3\left(1 - \frac{1}{x}\right);$$

$$3) \log_5(3x+1) + \log_5\left(\frac{1}{72x^2} + 1\right) \geq \log_5\left(\frac{1}{24x} + 1\right);$$

$$4) \log_3(2x+1) + \log_3\left(\frac{1}{32x^2} + 1\right) \geq \log_3\left(\frac{1}{16x} + 1\right);$$

$$5) \log_2(4x^2 - 1) - \log_2 x \leq \log_2\left(5x + \frac{9}{x} - 11\right);$$

$$6) \log_5(4x^2 - 9) - \log_5 x \leq \log_5\left(5x + \frac{5}{x} - 9\right).$$

Задание 12. Решите неравенство:

$$1) \log_{11}(8x^2+7) - \log_{11}(x^2+x+1) \geq \log_{11}\left(\frac{x}{x+5} + 7\right);$$

$$2) \log_7(11x^2+10) - \log_7(x^2+x+1) \geq \log_7\left(\frac{x}{x+8} + 10\right);$$

$$3) 2\log_2(x\sqrt{5}) - \log_2\left(\frac{x}{1-x}\right) \leq \log_2\left(5x^2 + \frac{1}{x} - 2\right);$$

$$4) 2\log_7(x\sqrt{2}) - \log_7\left(\frac{x}{1-x}\right) \leq \log_7\left(8x^2 + \frac{1}{x} - 5\right);$$

$$5) 9\log_{12}(x^2 - 3x - 4) \leq 10 + \log_{12} \frac{(x+1)^9}{x-4};$$

$$6) 11\log_{11}(x^2+x-20) \leq 12 + \log_{11} \frac{(x+5)^{11}}{x-4};$$

$$7) \frac{\log_2(8x) \cdot \log_3(27x)}{x^2 - |x|} \leq 0;$$

$$8) \frac{\log_3(9x) \cdot \log_4(64x)}{5x^2 - |x|} \leq 0;$$

$$9) \frac{\log_x(2x^{-1}) \cdot \log_x(2x^2)}{\log_{(2x)} x \cdot \log_{(2x^{-2})} x} < 40;$$

$$10) \frac{\log_x(5x^{-1}) \cdot \log_x(5x^3)}{\log_{(5x)} x \cdot \log_{(5x^{-3})} x} < 105$$

Задание 13. Решите неравенство:

$$1) \log_{49}(x+4) + \log_{(x^2+8x+16)} \sqrt{7} \leq -\frac{3}{4};$$

$$2) \log_{25}(x+3) + \log_{(x^2+6x+9)} \sqrt{5} \leq -\frac{3}{4};$$

$$3) \log_{16}(x+5) + \log_{(x^2+10x+25)} 2 \geq \frac{3}{4};$$

$$4) \log_{81}(x+6) + \log_{(x^2+12x+36)} 3 \geq \frac{3}{4};$$

$$5) (2-3x) \cdot \log_{2x-1}(x^2-2x+2) \leq 0;$$

$$6) (20-11x) \cdot \log_{5x-9}(x^2-4x+5) \leq 0;$$

$$7) (3x+10) \cdot \log_{2x+7}(x^2+6x+10) \geq 0;$$

$$8) (4x+9) \cdot \log_{2x+5}(x^2+4x+5) \geq 0.$$